NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 7
COURSE CODE: CAN702S	**COURSE NAME:** COMPLEX ANALYSIS
SESSION: JANUARY 2020	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	PROF. G. HEIMBECK
MODERATOR:	PROF. F. MASSAMBA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

Question 1 [11 marks]

Let U be a one-dimensional subspace of the $\mathbb{R}$-vector space $\mathbb{C}$ and $a \in \mathbb{C}$.

a) Prove that U is uniquely determined by the coset $a + U$. [4]

b) Show that the representatives of $a + U$ are exactly the elements of $a + U$. [5]

c) Prove that any two cosets of $a + U$ are disjoint or equal. [2]

Question 2 [13 marks]

a) What are separated subsets of $\mathbb{C}$? Which subsets of $\mathbb{C}$ are connected? State the definitions. [5]

b) Let $c, d \in \mathbb{C}$. You are reminded that the line segment with endpoints c and d is the set

$$\langle c, d \rangle := \{(1-\lambda)c + \lambda d \mid 0 \leq \lambda \leq 1\}.$$

Explain why $\langle c, d \rangle$ is connected. Proofs are not required. [5]

c) Let $D \subset \mathbb{C}$. Assume that there exists some $a \in D$ such that $\langle a, z \rangle \subset D$ for all $z \in D$. Show that D is connected. [3]

Question 3 [12 marks]

a) What is an argument of a non-zero complex number? State the definition. [2]

b) Let $z \in \mathbb{C}^{\times}$ and let $\varphi := \operatorname{Arg} z$ be the principal argument of z.

i) Show that $\operatorname{Re} z = |z| \cos \varphi$. [2]

ii) When is

$$\varphi = \arccos \frac{\operatorname{Re} z}{|z|}$$

true? State your reasons. [8]

Question 4 [13 marks]

Let $X \subset \mathbb{C}$, $f : X \to \mathbb{C}$ a function and $a \in X$.

a) Prove that f is continuous at a if and only if for each $\varepsilon > 0$, there exists some δ such that $f(N_\delta(a) \cap X) \subset N_\varepsilon(f(a))$. [3]

b) Let $(z_n) \in \mathbb{N}$ be a sequence in X which converges to a. If f is continuous at a, prove that $(f(z_n))_\mathbb{N}$ is convergent and $\lim\limits_{n\to\infty} f(z_n) = f(a)$. [4]

c) If f is not cintinuous at a, prove that there exists a sequence $(w_n)_\mathbb{N}$ in X which coverges to a but $(f(w_n))_\mathbb{N}$ does not converge to $f(a)$. [6]

Question 5 [18 marks]

a) When does the path integral $\int\limits_\gamma f(\zeta)\, d\zeta$ exist? Explain! [5]

b) Let γ be a continuously differentiable path. If $\int\limits_\gamma f(\zeta)\, d\zeta$ exists, show that $\int\limits_{-\gamma} f(\zeta)\, d\zeta$ exists and

$$\int\limits_{-\gamma} f(\zeta)\, d\zeta = -\int\limits_{\gamma} f(\zeta)\, d\zeta.$$

[5]

c) Let α and β be two paths in $\mathbb{C}$. When does $\alpha + \beta$ exist? State the definition and show that $\alpha + \beta$ is a path. [8]

Question 6 [17 marks]

a) State and prove the addition theorem of the exponential function. [5]

b) Let $\exp : \mathbb{C} \to \mathbb{C}$ be defined by $\exp(z) := e^z$. Show that exp is a homomorphism from the additive group $\mathbb{C}^+$ onto the multiplicative group $\mathbb{C}^\times$ of the field $\mathbb{C}$. [6]

c) What is a period of the function exp? Show that the periods of exp form the subgroup $\langle 2\pi i \rangle$ of $\mathbb{C}^+$. [6]

Question 7 [16 marks]

a) In comples analysis, what is an analytic function? State the definition. [4]

b) Show that every analytic function is a holomorphic function. [5]

c) Prove that every holomorphic function is an analytic function. [7]

End of the question paper